The Carrier Concentration Distributions and Recombination Rate at the p-n junction
ECE 435 Term Project

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1 Introduction

In this paper we study the coupled differential equations describing the current continuity for electrons and holes in a p-n junction. We use three different approaches, namely the use of the classical approximation for the recombination rate, leading to decoupled differential equations, the use of another familiar approximation for the recombination rate and finally the use of the exact expression for the recombination rate. The latter 2 cases lead to coupled differential equations which are solved after they are decoupled and linearized.

We will consider a one-dimensional symmetric p-n junction (Figure 1). We know that the equations of current continuity for electrons and holes and Gauss equation are:

$$\frac{\partial J_n}{\partial x} - e\frac{\partial n}{\partial t} = eU_s, \qquad \frac{\partial J_p}{\partial x} + e\frac{\partial p}{\partial t} = -eU_s, \qquad \frac{\partial \epsilon\epsilon_0 F}{\partial x} = e(p - n + N_D - N_A) \quad (1)$$

where $n(x)$ is the concentration of electrons, $p(x)$ the concentation of holes, $F(x)$ is the Electric Field, N_A and N_D are the concentarations of acceptors and donors respectively. $J_n(x)$ and $J_p(x)$ are the current densities for electrons and holes, given by the relations:

$$J_n = e\mu_n nF + eD_n\frac{\partial n}{\partial x}, \qquad J_p = e\mu_p pF - eD_p\frac{\partial p}{\partial x}, \quad (2)$$

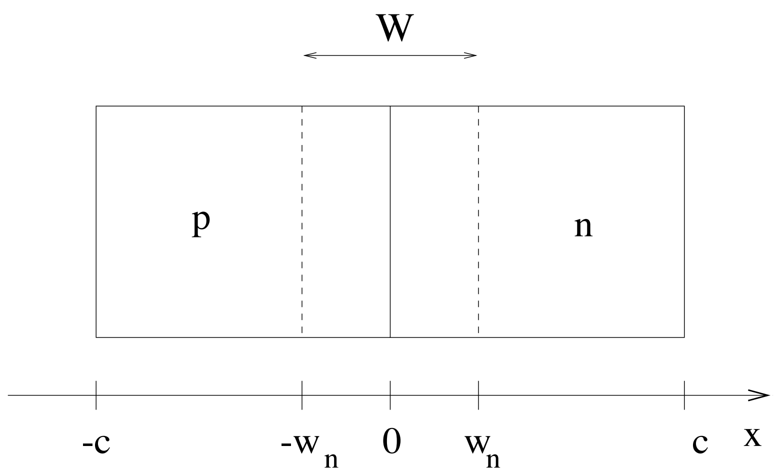

Figure 1: The linear and symmetric p-n junction

where μ_n, μ_p the mobilities and D_n, D_p the diffusion constants for the electrons and holes respectively. The recombination rate U_s is given by the relation:

$$U_s = \frac{np - n_i^2}{\tau_p(n + n_i) + \tau_n(p + n_i)} \tag{3}$$

We will try to solve the coupled equations (1), calculate the electron and hole concentrations and substitute in equation (3). The integral of the recombination rate in the depletion region and the base of the diode will give us the current density due recombination. We will study only the n-half of the diode, without loss of generality.

2 The Classical Approach

If we study the DC case (partial derivatives with respect to time are zero) and the base of the transistor ($F \simeq 0$) we arrive, using equations (1) and (2), to the coupled differential equations:

$$D_n \frac{\partial^2 n}{\partial x^2} = U_s, \qquad D_p \frac{\partial^2 p}{\partial x^2} = U_s, \tag{4}$$

One of the most common approximations applied, in order to decouple and simplify the equations (4), is to suppose that:

$$U_s \simeq \frac{(p - p_{no})}{\tau_p} \simeq \frac{(n - n_{no})}{\tau_n} \tag{5}$$

where p_{no}, n_{no} are the concentrations of holes and electrons respectively at the terminal, which is at position x=c (Figure 1). We realize that the equations decouple and become:

$$D_n \frac{\partial^2 n}{\partial x^2} = \frac{n - n_{no}}{\tau_n}, \qquad D_p \frac{\partial^2 p}{\partial x^2} = \frac{p - p_{no}}{\tau_p}, \tag{6}$$

The solutions of equations (6) are:

$$p_1^{base}(x) = (p_n - p_{no}) \frac{\sinh\left(\frac{c-x}{L_p}\right)}{\sinh\left(\frac{c-w_n}{L_p}\right)} + p_{no}, \tag{7}$$

and

$$n_1^{base}(x) = (n_n - n_{no}) \frac{\sinh\left(\frac{c-x}{L_n}\right)}{\sinh\left(\frac{c-w_n}{L_n}\right)} + n_{no}, \tag{8}$$

where $p_n \equiv p(w_n)$, $p_{no} \equiv p(c)$, $L_p = \sqrt{D_p \tau_p}$ (diffusion length), with equivalent formulas for the n-quantities. As we see in Figure 1, the depletion region is from 0 to w_n and the base of the diode is from w_n to c. We can now get an expression for $U_s(x)$ using equations (5) and (7). We get:

$$U_s^{base}(x) = \frac{(p_n - p_{no})}{\tau_p} \frac{\sinh\left(\frac{c-x}{L_p}\right)}{\sinh\left(\frac{c-w_n}{L_p}\right)} \tag{9}$$

The integral of the recombination rate (9) from w_n to c will give us the current density due recombination in the n base:

$$J_{recomb}^{base} = e \int_{wn}^{c} U_s^{base}(x) dx = e \frac{2L_p}{\tau_p} \frac{(p_n - p_{no})}{\sinh\left(\frac{c-w_n}{L_p}\right)} \sinh^2\left(\frac{c - w_n}{L_p}\right) \tag{10}$$

Let us now treat the depletion region ($0 \le x \le w_n$). According to the literature, we can obtain an accurate value of the hole concentration, if we simply set the sum of of the hole drift and diffusion currents equal to zero ($J_p = 0$). From equation (2) we get the first order differential equation:

$$eD_p \frac{\partial p}{\partial x} = e\mu_p pF, \qquad eD_n \frac{\partial n}{\partial x} = -e\mu_n nF,$$

and since, according to Einstein's relation $D_p = kT\mu_p/e$ and $D_n = kT\mu_n/e$, we get:

$$\frac{\partial p}{\partial x} = \frac{e}{kT} pF, \qquad \frac{\partial n}{\partial n} = -\frac{e}{kT} nF, \tag{11}$$

If V_{bi} is the built-in junction potential and V_j the externally applied one, we may approximate the Field in the depletion region as:

$$F = \frac{2(V_{bi} - V_j)}{W w_n}(x - w_n) \tag{12}$$

Equations (11) and (12) give me a faithful expression for the concentration of holes and electrons at the depletion:

$$p_1^{depl}(x) = p_n \exp\left[\frac{e}{kT}\frac{V_{bi} - V_j}{W w_n}(x - w_n)^2\right] \tag{13}$$

and

$$n_1^{depl}(x) = n_n \exp\left[-\frac{e}{kT}\frac{V_{bi} - V_j}{W w_n}(x - w_n)^2\right] \tag{14}$$

Now, since we have an acceptable relation for the concentration of holes in the depletion region, we return to our approximation for the recombination rate (relation (5)) and get the recombination rate for the depletion:

$$U_s^{depl}(x) = \frac{1}{\tau_p}\left(p_n \exp\left[\frac{e}{kT}\frac{V_{bi} - V_j}{W w_n}(x - w_n)^2\right] - p_{no}\right) \tag{15}$$

The current density due to recombination in the depletion region, is the integral of the equation above:

$$J_{recomb}^{depl} = e\int_0^{w_n} U_s^{depl}(x)dx = -e\frac{p_{no}}{\tau_p}w_n + \frac{\sqrt{\pi}}{2}\frac{e p_n}{\tau_p\sqrt{\kappa}}\mathrm{erf}(\sqrt{\kappa}w_n) \tag{16}$$

where,

$$\kappa \equiv \frac{e}{kT}\frac{(V_{bi} - V_j)}{W w_n} \tag{17}$$

We conclude that in the classical approach we can obtain exact expressions for the recombination current densities in the base and the depletion regions.

3 The Semiexact Approach

Now we will try to be more accurate with respect to the approximation we use for the recombination rate. We will use an approximation which is closer to the exact relation (3):

$$U_s(x) \simeq \frac{n(x)p(x) - n_i^2}{\xi} \tag{18}$$

where ξ is considered constant, given by:

$$\xi = \tau_p(n_{no} + n_i) + \tau_n(p_{no} + n_i) \tag{19}$$

Comparing equations (18) and (19) with equation (3) we realize that our assumption is that $n \simeq n_{no}$ and $p \simeq p_{no}$. We call it "semiexact" since it is closer to the exact relation for U_s.

Let us treat the base first. The field can be considered zero, so equations (1) and (2) transform, with the help of equation (18) into:

$$D_n \frac{\partial^2 n}{\partial x^2} = \frac{n(x)p(x) - n_i^2}{\xi}, \qquad D_p \frac{\partial^2 p}{\partial x^2} = \frac{n(x)p(x) - n_i^2}{\xi} \tag{20}$$

This is the coupled differential equations we have to solve. We first notice that:

$$D_n \frac{\partial^2 n}{\partial x^2} = D_p \frac{\partial^2 p}{\partial x^2}$$

which leads to the useful relation:

$$n(x) = \frac{D_p}{D_n} p(x) - \frac{Kx}{D_n} - \frac{\Lambda}{D_n} \tag{21}$$

where K, Λ are constants to be determined by the boundary conditions in the diode. Actually if one applies equation (21) twice using the sets of parameters: $\{x = w_n,\ p(w_n) = p_n \text{ and } n(w_n) = n_n\}$ and $\{x = c,\ p(c) = p_{no} \text{ and } n(c) = n_{no}\}$, one gets the following expressions for K and Λ:

$$K = \frac{(D_p p_{no} - D_n n_{no}) - (D_p p_n - D_n n_n)}{c - w_n} \tag{22}$$

and

$$\Lambda = \frac{-w_n(D_p p_{no} - D_n n_{no}) + c(D_p p_n - D_n n_n)}{c - w_n} \tag{23}$$

Substituting equation (21) into equation (20) we get an decoupled differential equation for $p(x)$ which is unfortunately non-linear:

$$\frac{\partial^2 p}{\partial x^2} = \frac{p^2 - Kxp/D_p - \Lambda/D_p - n_i^2 D_n/D_p}{\xi D_n} \equiv F(p, x) \tag{24}$$

A reasonable way to proceed is to linearize the equation above , by setting:

$$p = p_{no} + \delta p \qquad n = n_{no} + \delta n, \qquad x = c + \delta x \tag{25}$$

We realize that δp is positive and that δn and δx are negative. Using equations (24) and (25) we can write:

$$\frac{\partial^2 \delta p}{\partial (\delta x)^2} = U_o + A\delta p + B\delta x, \tag{26}$$

where

$$U_o = F(c) = U_s(c)/D_p = 0 \tag{27}$$

since $p(c)n(c) = p_{no}n_{no} = n_i^2$ and

$$A = \left.\frac{\partial F(p,x)}{\partial p}\right|_{\text{at } x = c} = \frac{2p_{no} - Kcp_{no}/D_p - \Lambda/D_p}{\xi D_n} \tag{28}$$

and

$$B = \left.\frac{\partial F(p,x)}{\partial x}\right|_{\text{at } x = c} = \frac{Kp_{no}}{D_p D_n \xi} \tag{29}$$

Equation (26) is now linear, so we can solve it easily (by first finding a general solution to the homogeneous equation and a special solution to the non-homogeneous one). The result is:

$$\delta p = p - p_{no} = a_1 \sinh(\sqrt{A}\delta x) + a2 + M\delta x \tag{30}$$

where $M = -B/A$ and a_1, a_2 are given by the boundary conditions. Actually, since $\delta p(c) = 0$ and $\delta p(w_n) = p_n - p_{no}$, we finally get:

$$p_2^{base}(x) = p_{no} + \frac{p_n - p_{no} - M(c - w_n)}{\sinh(\sqrt{A}(c - w_n))}\sinh(\sqrt{A}(c - x)) + M(c - x) \tag{31}$$

From equations (21) and (31) we get the solution for n(x):

$$n_2^{base}(x) = \frac{D_p}{D_n}\frac{p_n - p_{no} - M(c - w_n)}{\sinh(\sqrt{A}(c - w_n))}\sinh(\sqrt{A}(c-x)) + (\frac{D_p}{D_n}M)(c-x) - \frac{K}{D_n}x + (\frac{p_{no}D_p}{D_n} - \frac{\Lambda}{D_n}) \tag{32}$$

Of course we could have solved the linear equation for $n(x)$ and then use equation (21) to calculate $p(x)$. The choice depends on which concentration has values almost equal to the terminal ones through out the base and it is under investigation.

Using equations (31) and (32) above and equation (18) we can get a new expression for U_s^{base}:

$$U_s^{base}(x) \simeq \left(n_2^{base}(x)p_2^{base}(x) - n_i^2\right)/\xi =$$

$$Q + W\sinh^2(\sqrt{A}(c-x)) + E\sinh(\sqrt{A}(c-x)) + R\sinh(\sqrt{A}(c-x))x + Tx + Yx^2 \quad (33)$$

where:

$$Q = \left(\frac{D_p}{D_n}p_{no}^2 - \Lambda p_{no} + M^2c^2 + 2\frac{D_p}{D_n}p_{no}Mc - \Lambda c - n_i^2\right)/\xi \quad (34)$$

$$E = \left(\frac{D_p}{D_n}\Gamma^2\right)/\xi \quad (35)$$

$$R = \left(2\frac{D_p}{D_n}p_{no}\Gamma Mc - \Lambda\Gamma\right)/\xi \quad (36)$$

$$T = \left(2\frac{D_p}{D_n}p_{no}M - Kp_{no} - KMc - 2M^2c\right)/\xi \quad (37)$$

$$Y = \left(KM + M^2\right)/\xi \quad (38)$$

and

$$\Gamma = \frac{p_n - p_{no} - M(c - w_n)}{\sinh(\sqrt{A}(c - w_n))} \quad (39)$$

The integral of the recombination rate (33) from w_n to c will give us the current density due recombination in the n base:

$$\begin{gathered}
J_{recomb}^{base} = e\int_{wn}^{c} U_s^{base}(x)dx = \\
Q(c-x) + W\left(-(c-w_n)/2 + sinh(2\sqrt{A}(c-w_n))/(4\sqrt{A}\right) \\
+E\left(\tfrac{-1}{\sqrt{A}}(1 - \cosh(\sqrt{A}(c-w_n)))\right) \\
+R\left(-\tfrac{c}{\sqrt{A}} + \sqrt{A}w_n\cosh(\sqrt{A}(c-w_n)) - \tfrac{1}{A}\sinh(\sqrt{A}(c-w_n))\right) \\
+T(c^2/2 - w_n^2/2) + Y(c^3/3 - w_n^3/3)
\end{gathered} \quad (40)$$

Let us now proceed with the depletion region. If we follow the same principle of drift-diffusion cancelation, we get the same concentrations with the classical approach (equations (13) and (14)). If we use formula (18) for the recombination rate, we get,for the depletion:

$$U_s^{depl}(x) = \frac{p_n n_n - n_i^2}{\xi} \tag{41}$$

So the depletion current density due to recombination in the n-base becomes:

$$J_{recomb}^{depl} = e\int_0^{w_n} U_s^{depl} = e w_n \frac{p_n n_n - n_i^2}{\xi} \tag{42}$$

4 The Exact Approach

Now let us use the exact relation for $U_s(x)$ (equation(3)). The coupled differential equations become:

$$D_n \frac{\partial^2 n(x)}{\partial x^2} = \frac{n(x)p(x) - n_i^2}{\tau_p(n(x)+n_i) + \tau_n(p(x)+n_i)}, \qquad D_p \frac{\partial^2 p(x)}{\partial x^2} = \frac{n(x)p(x) - n_i^2}{\tau_p(n(x)+n_i) + \tau_n(p(x)+n_i)} \tag{43}$$

Equation (21) is still valid, so we can write the following dicoupled equation for p(x):

$$D_p \frac{\partial^2 p}{\partial x^2} = \frac{(p/D_n - Kx/D_n/D_p - \Lambda/D_n/D_p)p - n_i^2/D_p}{\tau_p((D_p p/D_n - Kx/D_n - \Lambda/D_n + n_i) + \tau_n(p+n_i)} \equiv F'(p,x) \tag{44}$$

We can now again realize the last equation, by using relations (25). We then get the linear equation:

$$\frac{\partial^2 \delta p}{\partial (\delta x)^2} = U_o' + A'\delta p + B'\delta x, \tag{45}$$

where

$$U_o' = F'(c) = U_s(c)/D_p = 0 \tag{46}$$

since $p(c)n(c) = p_{no}n_{no} = n_i^2$ and

$$A' = \left.\frac{\partial F(p,x)}{\partial p}\right|_{\text{at } x=c} =$$

$$\frac{(2D_p p/D_n - Kx/D_n - \Lambda/D_n)\mathcal{D} - (D_p p^2/D_n - Kpx/D_n - \Lambda p/D_n - n_i^2)(D_p^2\tau_p/D_n + D_p\tau_n)}{\mathcal{D}^2} \tag{47}$$

and

$$B' = \frac{\partial F(p,x)}{\partial x}\bigg|_{\text{at } x=c} = \frac{(Kp/D_n)\mathcal{D} + (D_p p^2/D_n - Kpx/D_n - \Lambda p/D_n - n_i^2)(KD_p/D_n\tau_p)}{\mathcal{D}^2} \tag{48}$$

where

$$\mathcal{D} = \tau_p((D_p p_{no}/D_n - Kc/D_n - \Lambda/D_n + n_i) + \tau_n(p_{no} + n_i) \tag{49}$$

Now we can easily solve the linearized equation (45) and apply the boundary conditions to calculate the arising constants. The solution is the equivalent of equations (31):

$$p_3^{base} = p_{no} + \frac{p_n - p_{no} - M'(c - w_n)}{\sinh(\sqrt{A'}(c - w_n))}\sinh(\sqrt{A'}(c-x)) + M'(c-x) \tag{50}$$

From equations (21) and (50) we get the solution for n(x):

$$n_3^{base} = \frac{D_p}{D_n}\frac{p_n - p_{no} - M'(c - w_n)}{\sinh(\sqrt{A'}(c - w_n))}\sinh(\sqrt{A'}(c-x)) + (\frac{D_p}{D_n}M')(c-x) - \frac{K}{D_n}x + (\frac{p_{no}D_p}{D_n} - \frac{\Lambda}{D_n}) \tag{51}$$

where $M' = -B'/A$. Using equations (50) and (51) above and equation (3) we can get a new expression for U_s^{base}:

$$U_s^{base}(x) = \frac{n_3^{base}(x)p_3^{base}(x) - n_i^2}{\tau_p(D_p p_3^{base}(x)/D_n - Kx/D_n - \Lambda/D_n + n_i) + \tau_n(p_3^{base}(x) + n_i)} \tag{52}$$

$$J_{recomb}^{base} = e\int_c^{w_n} \frac{n_3^{base}(x)p_3^{base}(x) - n_i^2}{\tau_p(D_p p_3^{base}(x)/D_n - Kx/D_n - \Lambda/D_n + n_i) + \tau_n(p_3^{base}(x) + n_i)}dx \tag{53}$$

It seems that it is difficult to calculate the analytic form of the integral of (52). But we can always calculate it numerically and compare the result with other approximations. Another approach is to consider the denominator of equation (52) constant ($\simeq \xi$). Then we can proceed to the integration and get a result equivalent to the semiexact case.

Now, for the depletion region we just use equations (13) and (14). The recombination current becomes:

$$J_{recomb}^{depl} = e\int_0^{w_n} \frac{n_1^{depl}(x)p_1^{depl}(x) - n_i^2}{\tau_p(D_p p_1^{depl}(x)/D_n - Kx/D_n - \Lambda D_n + n_i) + \tau_n(p_1^{depl}(x) + n_i)}dx \tag{54}$$

which again is difficult to calculate analytically, but numerical methods can be incorporated.

5 Numerical Example and Comparison

We now compare the results we obtained in the previous three sections by using a numerical example. The values we use for our physical quantities are:

$$c = 5 \cdot 10^{-4} \cdot \quad \mathrm{cm}$$

$$n_i = 7.82 \cdot 10^9 \quad \mathrm{cm}^{-1}$$

$$D_p = 26 \quad \mathrm{cm}^2/\mathrm{sec}$$

$$D_n = 26 \quad \mathrm{cm}^2/\mathrm{sec}$$

$$n_{no} = 10^{14} \quad \mathrm{cm}^{-1}$$

$$p_{no} = n_i^2/n_{no} = 611524 \quad \mathrm{cm}^{-1}$$

$$V_{bi} = 0.727 \quad \mathrm{V}$$

$$V_j = 0.25 \quad \mathrm{V}$$

$$p_n = p_{no} \cdot e^{eV_j/kT} = p_{no} \cdot e^{V_j/8.617/10^{-5}/300} = 9.69 \cdot 10^9 \quad \mathrm{cm}^{-1}$$

$$n_n = n_{no} = 10^{14} \quad \mathrm{cm}^{-1}$$

$$W = 10.23 \cdot 10^{-6} \quad \mathrm{cm}$$

$$w_n = W/2 = 5.13 \cdot 10^{-6} \quad \mathrm{cm}$$

$$tn = 10^{-11} \quad \mathrm{sec}$$

$$tp = 10^{-11} \quad \mathrm{sec}$$

Using equations (10), (40) and (53) we get the following solutions for the base current density due to recombination:

$$J^{base}_{recomb} = 1.56261 \cdot 10^{16} e = 2.5033 \quad \text{mA} \qquad \text{If we use the Classical approach}$$
$$J^{base}_{recomb} = 2.0025 \cdot 10^{15} e = 0.3 \quad \text{mA} \qquad \text{If we use the Semiexact approach}$$
$$J^{base}_{recomb} = 1.56241 \cdot 10^{16} e = 2.5029 \quad \text{mA} \qquad \text{If we use the Exact approach}$$

For the depletion region current densities due to recombination we get (using equations (16),(42) and (54)):

$$J^{depl}_{recomb} = 2.92383 \cdot 10^{18} e = 0.468 \quad \text{A} \qquad \text{If we use the Classical approach}$$
$$J^{depl}_{recomb} = 4.975 \cdot 10^{15} e = 0.79 \quad \text{mA} \qquad \text{If we use the Semiexact approach}$$
$$J^{depl}_{recomb} = 6.35943 \cdot 10^{16} e = 10.19 \quad \text{mA} \qquad \text{If we use the Exact approach}$$

We see that for this particular example, the semiexact approach underestimates the current density of the base, but it is closer to the exact value in the depletion region. The classical approximation is satisfactory for the base of the diode, but not for the depletion region. Actually, since there is not satisfactory consistency between the three methods for the depletion region, we have to investigate further the validity of our recombination rate approximations.

6 Conclusions

In this paper we tried to calculate explicitly and analytically the carrier concentrations and current densities of the base and depletion regions of the diode. We described the classical approximations and developed solutions for the coupled differential equations describing the system, for the semiexact and exact case (refering to the expression for the recombination current). We described a particular arithmetic example and noticed different results, for the three approaches. The question of how the applied voltage and the approximations incorporated influence the different approaches is under investigation.

7 References

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